

# Sub-Nyquist Cognitive Radio System

Etgar Israeli, Shahar Tsiper, Deborah Cohen, Eli Shoshan, Alex Reysensen, Yonina C. Eldar  
Technion - Israel Institute of Technology, Haifa, Israel

## Abstract

*We demonstrate of a real-time sub-Nyquist sampling and reconstruction system, based on prototype hardware and an embedded proprietary card, the sub-Nyquist Cognitive Radio setup (CogRadio). Our system detects the support and reconstructs a wide-band sparse signals with arbitrary support from sub-Nyquist samples. This process takes place in real-time. The CogRadio system is shown to comply with cognitive radios requirements.*

## 1 Introduction and Motivations

In light of the ever-increasing demand for new spectral bands and the under-utilization of those already allocated, the concept of Cognitive Radio (CR) has emerged [1]. Opportunistic users could exploit temporarily vacant bands after detecting the absence of activity of their owners. A crucial task in the CR cycle is therefore spectrum sensing and detection which has to be precise and efficient. Yet, CRs typically deal with wideband signals whose Nyquist rates are very high. Several sub-Nyquist sampling methods have recently been proposed [2], alleviating both the software and hardware burden.

We present a hardware demo of such a method, the modulated wideband converter (MWC) [2], adapted to a CR scenario. A multiband input signal of Nyquist rate  $f_{\text{Nyq}} = 6\text{GHz}$  with total bandwidth occupancy of 120MHz is sampled at an overall sampling rate of 360 MHz - only 5.3% of the Nyquist rate. We show signal reconstruction in a blind setting, namely the carrier frequencies of the transmissions are unknown. For CR purpose, power spectrum reconstruction suffices and can be achieved at a lower sampling rate. The CR abilities that are demonstrated here surpass the IEEE 802.22 CR standard [1] requirements that considers Nyquist rates below 1 GHz.

The Nyquist rate of the input signal is significantly higher than those considered in previous works. This leads to considerable challenges in terms of hardware. One of the main contributions of this implementation

is the calibration process which is performed once offline. The demo then runs in real time.

To summarize, the input,  $x(t)$ , is an arbitrary wide-band RF signal, with one constraint - support sparsity. Our output is a complete reconstruction of the signal, while using only a fraction of Nyquist rate, for the sampling and digital processing.

## 2 Scientific and Technical Description

Our system consists of two main parts: Analog Front-End - receives the wide-band input RF signal, and generates  $p = 3$  mixed and low-passed signals, as seen on the left of Fig. 1. Digital Back-End - receives  $p = 3$  mixed inputs, samples them at the very low  $f_s = 120\text{MHz}$  rate, as seen on the right of Fig. 1.

### 2.1 Analog Front-End (CogRadio Card)

An analog mixing front-end aliases the spectrum, such that a spectrum portion from each band appears in base-band. The system consists of  $p$  parallel channels. In the  $i$ -th channel, the input signal  $x(t)$  is multiplied by a mixing function  $p_i(t)$  which is  $T_p$  periodic. The mixing functions  $p_i(t)$  are chosen as a piecewise constant functions that alternates between the levels  $\pm 1$  for each of  $M$  equal time intervals.

After mixing, the signal spectrum is truncated by a low-pass filter with cutoff  $1/(2T_s)$  and the filtered signal is sampled at rate  $1/T_s$ . The sampling rate of each channel is sufficiently low, so that existing commercial analog to digital converters (ADCs) can be used for that task.

The unknown spectrum of  $x(t)$ , namely  $X(f)$ , can be related to the known DTFTs of the samples  $\mathbf{y}(f)$  such that

$$\mathbf{y}(f) = \mathbf{A}\mathbf{z}(f), \quad f \in [-f_s/2, f_s/2], \quad (1)$$

where  $\mathbf{z}(f)$  contains slices of  $X(f)$  of length  $f_s$ . The matrix  $\mathbf{A}$  contains the Fourier series coefficients of the mixing functions  $p_i(t)$ .

The design parameters are therefore the number of channels  $p$ , the expansion factor  $q$  (explained in section 2.2.1), the period  $T_p$ , and the sampling rate

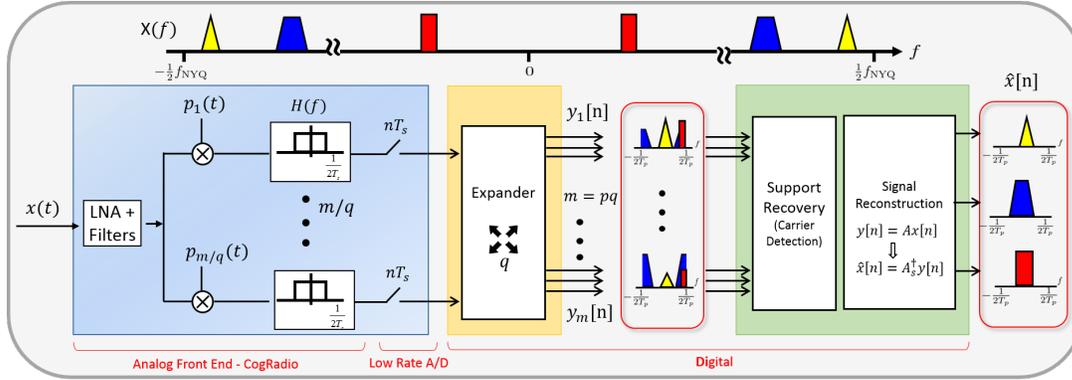


Figure 1. CogRadio Analog System and Digital Processing

$1/T_s$ . In our demo,  $p = 3$ ,  $q = 5$ ,  $f_p = 20\text{MHz}$ , and  $f_s = 120\text{MHz}$ .

## 2.2 Digital Processing - Reconstruction

### 2.2.1 Expander

The burden on hardware implementation is highly affected by the total number of hardware devices, which includes mixers, low-pass filters and ADCs. Clearly, it would be beneficial to reduce the number of channels. To that purpose, we use the expander that reduces the number of channels at the expense of a higher sampling rate  $f_s \geq q \cdot f_p$  per channel and additional digital processing (see [2] for further details).

### 2.2.2 Support Detection and Reconstruction

The digital reconstruction algorithm can be divided into two stages: support recovery and signal recovery (in the case of power spectrum recovery, the sampling rate can be further reduced).

First, the support of the input signal is recovered from the samples by the continuous-to-finite (CTF) block, using compressed sensing techniques, adapted to analog signals [2]. The CTF outputs an index set  $S$  of active slices from the vector  $\mathbf{z}(f)$ . The system (1) is then reduced to the support and solved using the Moore-Penrose pseudo-inverse. The whole digital processing, after the expander, is performed at the low rates of  $f_p$ .

The whole system is shown in Fig. 2.

## 2.3 System Calibration

Although digital Matlab<sup>®</sup> simulations of the sub-Nyquist algorithms proved very successful in the past, actual implementation remained a challenge. Certain effects can have great impact on the sampling matrix  $\mathbf{A}$ , such as:

- Analog mixers behavior in our usage scenario. They were never designed to receive multiple sinusoids.
- Filters behavior. Previous systems suffered from the non-linear phase response in analog IIR filters.
- Phase noise and jitter, due to variations in components, cables and clock deltas.
- Effects of real signal-power to noise ratios.
- Syncing between the analog and digital parts.

Since no models exist for such effects, we first need to calibrate our system in order to find the actual sampling matrix  $\mathbf{A}$ , since reconstruction fails with the theoretical one.

To overcome these difficulties, we developed a calibration algorithm for the entire CogRadio system. This algorithm allows us to achieve precise support detection and reconstruction when using different analog components by compensating for those imprecisions and variations. As a result, many of the effects stated above, are inherently solved after successful calibration.

We use  $\lceil f_{\text{Nyq}}/2f_p \rceil = 153$  iterations for the calibration process performed once off-line. In each iteration we perform the following steps:

1. Inserting sine waves as input. The sine frequencies are  $f_p$  increments:  $f_k = k \cdot f_p + f_0$ ,  $k \in \{1.. \lceil f_{\text{max}}/f_p \rceil\}$ .
2. Measuring the output signal, that contains  $q = 5$  mixed sine waves, that are folded to baseband.
3. Estimating the sine coefficients: amplitude, frequency and phase. The estimation is done using a combination of spectrum estimation and least-squares optimization.
4. Using developed formulas on the sine coefficients, we find the coefficients of the matrix  $\mathbf{A}$ .

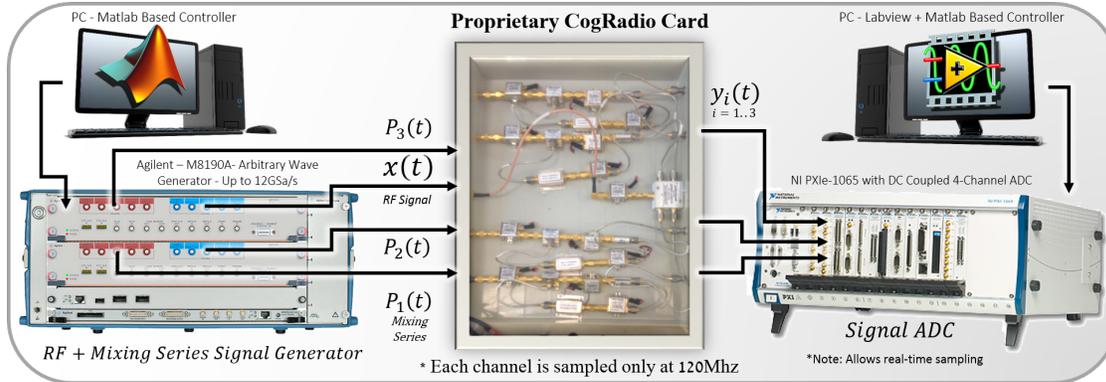


Figure 2. CogRadio System - Actual Equipment

### 3 Implementation and Use

The heart of the entire system is our proprietary developed CogRadio card. The RF input  $x(t)$  and the mixing series  $p_i(t)$  are generated using the arbitrary wave generator (AWG) - Agilent M8190.

The CogRadio card is implemented using connector based analog components, which allow us to tweak and modify the card characteristics.

The digital back-end is implemented using the National Instruments PXIe-1065 computer with DC coupled ADC. Very low computational load is required in order to achieve real time recovery and reconstruction, since all processing is done in the very low  $f_p$  rate. The matrix  $\mathbf{A}$  and its pseudo-inverse are calculated once, and then stored in memory, regardless of the input carrier frequencies. Matlab<sup>®</sup> and Labview<sup>®</sup> environments are used to simulate the various digital operations.

The exact system components used throughout the demo are shown in Figure 2. The specs used by the CogRadio card are given in Table 1.

During the demo, we will feed RF signals with Nyquist rate of 6GHz, total maximal bandwidth occupancy of 120 MHz, and varying support into the system. We'll present them in frequency and time domain, and compare them to the reconstructed output. Visitors will get a chance to see a useful and practical implementation of a sub-Nyquist CR, and understand its potential for future applications.

### 4 Conclusions and Future Developments

We demonstrate a real-time sub-Nyquist sampling and reconstruction system in practice, for signals from the multiband model. We can recover either the signal itself or only its power spectrum which suffices for

	Value	Notes
$f_s$	120 MHz	$(q + 1) f_p$ - Sampling rate
$f_p$	20 MHz	$1/T_p$
$q$	5	Expansion factor
$p$	3	# Hardware Channels
$f_{\max}$	3 GHz	$f_{\max} = f_{\text{Nyq}}/2$
$B$	19.5 MHz	Bandwidth on each carrier

Table 1. CogRadio Card Specs

detection purposes in CR. The main challenge of this hardware demo system is the calibration stage which is crucial for the reconstruction.

Our system deals with higher Nyquist rates than those considered in the IEEE 802.22 protocol for CR applications on TV bands, as seen in [1], yet we sample them at a very low rate, roughly 5% of Nyquist.

The next system we are currently working on is a collaborative spectrum sensing system where each CR samples an input signal suffering from fading and shadowing effects. The different CRs then jointly reconstruct the signal support from the sub-Nyquist samples in a distributive way.

### References

- [1] IEEE-Computer-Society. Cognitive wireless ran medium access control (mac) and physical layer (phy) specifications. *IEEE Standard-802.22*, July 2011.
- [2] M. Mishali and Y. C. Eldar. From theory to practice: Sub-Nyquist sampling of sparse wideband analog signals. *IEEE J. Sel. Top. Sig. Proc.*, 4(2):375–391, Apr. 2010.

Sub-Nyquist CogRadio Gallery

<http://goo.gl/WDB6Zl>